Preparedness Metrics for Dynamic Rescue Resource Planning

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ABSTRACT

Dynamic planning in fire and rescue services (FRS) relies on smaller, mobile units rather than traditional station-based deployment. A key challenge in this approach is maintaining adequate preparedness, ensuring the ability to respond effectively to new emergencies. This paper presents a quantitative preparedness measure tailored for FRS, developed using mathematical modeling and calibrated through expert input. The measure supports decision-making by evaluating the availability and response times of required resources. Validation with fire service personnel across Sweden revealed that the measure aligns with expert assessments in approximately 60% of cases, highlighting variability in subjective judgments. The findings underscore both the potential and limitations of standardized preparedness metrics and suggest directions for improving decision support in dynamic FRS planning.

Keywords

Accidents, Decision support systems, Emergency management, Heuristics, Integer programming.

INTRODUCTION

Fire and rescue services (FRS) have traditionally been characterized by relatively static planning and resource management. Fire stations are strategically located in populated areas and the firefighters wait at the station for an alert. There are several drawbacks with this planning, such as that the need for assistance varies during the day, or that the resources are not always present at the station, which makes it unlikely that resources are optimally located at all times.

Recently, a new more dynamic way of planning FRS has started to spread in Sweden. The firefighters are divided into smaller, more flexible units where some can perform accident preventive tasks while others are strategically located outside the station to maintain a good preparedness for responding to new accidents. When an accident occurs, the resources with the shortest response times are dispatched. If a small unit is dispatched, it might also be necessary to dispatch additional units to achieve full response, i.e. the set of resources required for effective work at the incident site.

This new dynamic planning and control strategy generates benefits such as shorter first response times, improved coverage and the possibility to do more work to prevent accidents. Dynamic planning comes with some increase in cost, as additional vehicles might be required, and the total travelled distance per year increases. This increase is generally considered insignificant though, since the major cost component for providing fire and rescue services is salaries. However, the complexity of the planning situation also increases. Instead of just keeping track of which resources that are available at the station, the decision makers must control a large number of small units spread over the area. Thus, it becomes more complicated to assess the preparedness in the area (i.e. to evaluate the current ability to respond to new accidents), and to make decisions about which resources to dispatch to new accidents or how to relocate resources to improve the preparedness. Quantifying and visualizing the preparedness can support this complex planning process.

In this work, we develop and validate a preparedness measure for FRS, which can be used in a decision support system for dynamic planning of FRS resources. To select which resources to include in the measure, we develop and solve an integer programming model both exactly and heuristically. We then compare the solutions obtained from the heuristic with the optimal solutions. Further, we present a method for calibrating the quantitative levels of the preparedness measure to match the expectations of FRS personnel. Finally, we experimentally compare the preparedness calculations, divided into three preparedness levels, with evaluations made by FRS personnel from emergency services across Sweden, to check the degree of agreement. This is done using operational scenarios from the C3Rescue system (http://www.c3learninglabs.com). The paper concludes with potential applications in decision support systems and outlines future research directions.

Previous related work

Preparedness measures assess an organization's (or country's or individual's) ability to respond to emergencies and disasters, balancing the probability of events with available response capability (Andersson Granberg, 2013). Various frameworks have been developed to capture this concept, especially in the context of major disasters. For example, Birkmann (2007) introduced multi-scale risk and vulnerability indicators; Novelo-Casanova and Suárez (2015) created a statistical Risk Management Index (RMI) focused on disaster preparedness; and Kusumastuti et al. (2014) developed a community disaster resilience index that incorporates local socio-economic factors. Recently, Vernaccini et al. (2025) introduced the Dynamic Preparedness Metric (DPM), a global and regional framework for assessing health emergency preparedness. These methods primarily address high-level, large-scale event preparedness, rather than the day-to-day realities of emergency services.

In contrast, daily emergency operations require measures that reflect rapidly changing conditions. Manca and Brambilla (2011) tackled this challenge in the context of road tunnel incidents, using an analytic hierarchy process to quantify emergency preparedness, but their metric was confined to tunnel scenarios and was relatively static. Yu et al. (2014) presented an indicator-based public safety index for urban areas to track safety trends, while Wallace (1983) developed an index for evaluating the adequacy of fire services in New York City. However, these indices were mainly retrospective, used for analyzing past system performance rather than guiding real-time operational decisions.

Andersson and Värbrand (2007) developed a preparedness index specifically for Emergency Medical Services (EMS). Their index quantified coverage by available ambulances, factoring in expected response times and spatial demand in each zone. Preparedness in a given area decreased if ambulances were farther away or if demand was high, and the overall preparedness of the system was determined by the least-covered zone. This index supported dynamic dispatch and relocation strategies, enabling decisions that maintained high preparedness system-wide rather than simply dispatching the nearest unit. Lee et al. built on this approach: Lee (2011) demonstrated that nearest-unit dispatch is not always optimal, and a preparedness-based policy can reduce mean response times. Lee (2017) introduced a refined preparedness metric that accounted for system centrality, further improving dispatch decisions. These studies confirmed that integrating preparedness measures into real-time EMS decision-making can yield tangible performance benefits, such as faster responses and improved coverage. Hall and Sekizawa (1991) contributed with a broader framework for fire risk analysis, linking hazard modeling with response capability, concepts integral to preparedness measurement for fire services.

Initially, operations research (OR) in emergency services focused on strategic or tactical planning. Early work, such as Toregas (1971) and Daskin (1983), addressed station location models, while Swersey's 1994 survey summarized models for siting stations and allocating crews. Operational, real-time planning, such as dynamic deployment, got less attention in the beginning. Exceptions include Swersey (1982), who used a Markov decision model to determine how many fire companies to dispatch to uncertain alarms, and Ignall et al. (1982), who developed an algorithm for initial fire company dispatch. These studies were among the first to examine dispatch rules beyond simple proximity. Later, Haghani and Yang (2007) incorporated real-time factors like traffic congestion into fire dispatch models, showing that rerouting and dynamic resource management can improve response times. Kolesar and Walker (1974) introduced a method for temporarily redeploying fire units when coverage dropped below a threshold—an idea that anticipated the dynamic ambulance relocation problem later tackled in EMS research. Aringhieri et al. (2017) provide a comprehensive review of dynamic EMS operations research, noting a shift in the 2010s toward strategies that integrated real-time relocation and dispatch algorithms using preparedness metrics.

Building on these foundations, recent studies have advanced optimization models for emergency resource management. Bélanger et al. (2019) reviewed post-2010 developments in ambulance location, relocation, and dispatch, highlighting the trend toward integrated and stochastic approaches that explicitly model preparedness and system state. Carvalho et al. (2020) further integrated EMS dispatch and relocation optimization, using a mixed-integer model based on a time-based preparedness metric, and demonstrated improved system-wide readiness and response times in Lisbon. In fire services, recent research has begun adapting these preparedness-centric methods. For instance, Pérez et al. (2016) optimized fire station configuration and fleet mix for Santiago, Chile, while Dahlgren et al. (2009) evaluated the use of mobile (non-stationary) fire units to improve coverage, noting both the benefits and increased complexity of readiness assessment. Park et al. (2016) deployed a railway crossing monitoring system to provide live blockage data to fire dispatchers, reducing response times by rerouting units in real time. Granberg (2022) proposed an optimization model for fire and rescue dispatch that applies preparedness-based principles from EMS, using integer programming and heuristics to select which units to send to an incident so as to maximize the preparedness remaining for future calls. These works reflect a growing trend of applying dynamic, preparedness-driven planning to fire services, an area that has historically lagged behind EMS.

Building on these, our work develops a tailored preparedness measure for FRS and demonstrates its use in dynamic planning. This contribution can be seen as an extension of the EMS models (Andersson & Värbrand, 2007; Lee, 2011) into the FRS domain, with adjustments for the complexity of fire incidents.

CONSTRUCTING A PREPAREDNESS MEASURE

The word preparedness is commonly used by FRS in Sweden. It is an expression of how well prepared the FRS are to deploy the required resources for a full response (e.g. to initiate smoke diving, there are certain requirements on resources; if anything is missing, the team has to wait). To effectively complete the response work, additional resources are usually deployed depending on the specifics of the situation. The preparedness measure however focuses on the minimum requirement for initiation of response. This means that we define a "full response" as the predetermined number of specific vehicles and personnel resources (competences) that will be immediately dispatched to a specific event. The preparedness also depends on how quickly all these required resources can reach the event site, i.e. the full response time.

Selection of events to include in the measure

For dynamic planning of FRS resources, it is central to evaluate the preparedness for frequent (daily) emergencies requiring an urgent response. Accidents that rarely occur or that require a large amount of resources are too costly to be continually prepared for. Thus, when dynamically planning the resource movements and allocations, the FRS in Sweden aim for rapid response to daily emergencies, like building fires and traffic accidents. Fires and traffic accidents represented more than 80% of all the responses made to accidents during 2024, not counting false alarms (https://www.mcf.se/sv/verktyg--tjanster/statistikverktyg-for-brander-och-olyckor/). These accident types also generate high costs in terms of lost lives, suffering and lost property value. Therefore, in this study, the events selected to measure the preparedness for were building fires, fires in high-rise buildings (since these require additional resources compared to building fires) and traffic accidents.

Indicators of preparedness

Preparedness has both objective and subjective facets. According to FRS in Sweden, the main indicators for estimating preparedness are 1) the expected response time for resources that are required for a full response to a specific accident, and 2) the expected number of accidents of the specific type in the area under consideration. For the preparedness to be good, the response time should be short where the demand is high, while it is possible to accept a longer response time to areas with a low demand. If the initial resources needed to handle a specific accident cannot reach the site at the same time, it is considered beneficial if some of them can get there quickly to perform a first response. The first responders might be able to perform cardiopulmonary resuscitation (CPR; chest compressions), first aid, extinguish small fires, open doors, warn and inform affected people and otherwise prepare for the full response.

All the required resources are essential for the response, and the FRS cannot say if it is preferable (in general) that a particular type of resource arrives quicker than another type. Thus, the average response time for the resources required for a full response is used as one indicator of the preparedness for that accident type. Another natural choice might be to focus on the response times for the first and last arriving resources, instead of the average. However, since the FRS experts clearly stated that a short response time is important for all resources, we opted to utilize the average response time.

Assume that the requested resources in the example in Figure 1 are one base unit, one ladder unit, one team leader, four basic firefighters, two ladder vehicle operators and one BA operator (a breathing apparatus operator can lead a team of firefighters into a burning building for life rescue and fire extinguishing). The resources contributing to the preparedness are the ones that can reach the site quickest, i.e. the two basic firefighters that can arrive within eight minutes; the team leader, two basic firefighters, the BA operator, and the base vehicle that can arrive within 10 minutes; and the remaining two required ladder vehicle operators and the ladder vehicle that an arrive within 14 minutes. Together they get an average response time of 10.8 minutes.

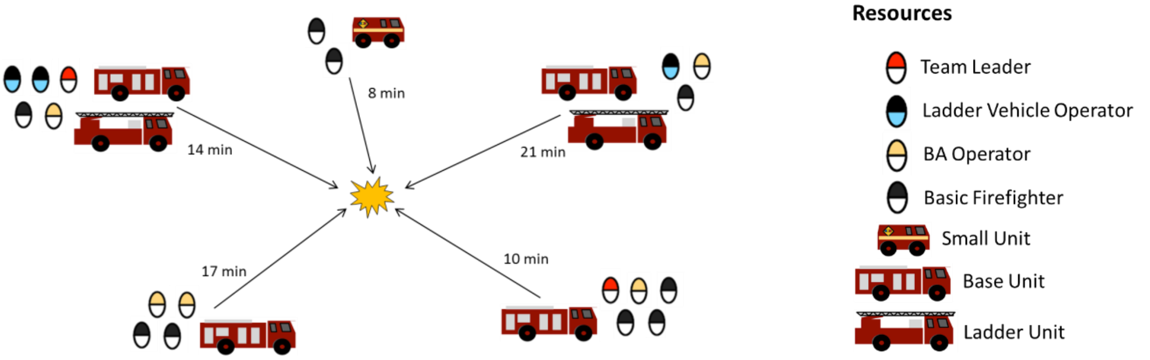

Figure 1. Example of how available resources can respond to an incident and contribute to the preparedness

Combining the preparedness indicators

The indicators are combined by multiplying them in a p-median fashion (Hakimi, 1965). Assume that accident type in Figure 1 has a frequency of 1.4 events per year, in the location under consideration. The expression to measure the preparedness for the specific location and accident type in Figure 1 then becomes:

$$P = 1.4\left(\frac{2*8+5*10+3*14}{10}\right) \approx 15.12$$

During the development of the measure, FRS experts found that the expected demand factor had too much impact on the preparedness in relation to the response time. Therefore, the demand component was modelled as the accident frequency taken to the power of γ, with a γ between 0 and 1. This can be seen as one way of ensuring some equity between areas with varying accident frequency, e.g. rural and urban areas. It thus reflects the opinions of the FRS experts, in balancing risk and response ability. After validation testing together with FRS experts, $\gamma = 0.25$ was used in the experiments, effectively downscaling the effect of the demand factor.

Formulation of the preparedness measure

The general expression to measure the preparedness for handling a particular accident of type a in a specific zone j is formulated as:

$$P_{aj} = \text{demand component } * \text{average response time of requested resources}$$

$$P_{aj} = \left(d_{aj}\right)^{\gamma}\left(\frac{\sum_{p\epsilon P} V_{paj} + \sum_{u\in U} T_{uaj}}{\sum_{r\in R} B_{ra} + \sum_{k\in K} D_{ka}}\right) \quad \forall\, a \in A, j \in J \qquad (1)$$

where V_{paj} is the response time for a requested personnel resource p that contributes to the preparedness for handling accidents of type a in zone j, T_{uaj} is the response time for a requested vehicle u, B_{ra} is the number of required personnel resources with a certain competence r and D_{ka} is the number of vehicles of type k that are needed for accidents of type a. Finally, d_{aj} is the expected number of accidents of type a in zone j.

In practice, the response times for personnel and vehicles are coupled, as personnel will use a vehicle to travel to the incident site. This means that there is an assignment problem that needs to be solved, deciding which person should travel with which vehicle (as can be seen in Appendix A). This can also be predetermined, but whichever the case, the response time for coupled resources will depend on the slowest resource, effectively setting the response times for all personnel traveling in the same vehicle, as well as for the vehicle itself, to the same value.

It should be noted that the preparedness measure is developed for operational use, i.e. the value of P_{aj} describes the current preparedness, given that a certain set of resources are available. The measure easily adapts to simultaneously occurring accidents, simply by not taking into account resources that become unavailable. The assumption that resources available now, also will be available in the future, is not applicable for high demand emergency service systems, like EMS. Thus, when evaluating preparedness in those systems, it is necessary to incorporate an availability (or busy) component in the measure. However, for most FRS in Sweden, the proportion of time spent on emergency response and work at the accident site is low. Thus, for our purposes, we believe that it is a reasonable assumption.

As you always prepare for the future, a preparedness measure should be based on expected events and expected response times. However, it is important to follow up how well the predictions of the included indicators match reality. In this case, it may include calculating errors for the accident forecast, and comparing real historical response times to the ones calculated by the measure.

While by construction, a low preparedness value is good, the value in itself does not express if the preparedness is good or inadequate. These qualitative interpretations are subjective and depend on type of accident, situation, FRS organization, response area or country. Therefore, to be able to use the measure, it must be thoroughly tested, calibrated and validated.

When calculating the preparedness, the contributing resource should be selected to minimize the response times V_{paj} and T_{uaj}. The resources may have multiple skills. A firefighter might for example both be able to operate in smoke filled spaces as well as manage ladder vehicles. The firefighter can however only utilize one skill at a time. Thus, it is not always obvious how the resources should be selected; looking at the example in Figure 1, the Team leader and the BA operator located 10 minutes away from the site also have the basic firefighting competence. It would thus be possible to select these resources for basic firefighting, which mean that the team leading and BA operating competence have to be obtained from the location 14 minutes away. To select the most suitable resources, a mathematical model is developed, and described in the following section.

A RESOURCE SELECTION MODEL

The resource selection model is a mixed integer programming model that minimizes the sum of the response times V_{paj} and T_{uaj}. Since the preparedness is evaluated for each accident type and zone, the model has to be solved for each $a \in A$ and $j \in J$. The model allocates staff to vehicles, and decides which competence that each person should use at the response site. The complete model is described in detail in Appendix A.

Solution method for the resource selection model

The model can be solved by Gurobi Optimizer 5.6.0, but the solution time is too long to be of practical value in an operational situation – typically it takes between one and ten hours to produce optimal solutions for reasonably sized scenarios. Therefore, a heuristic using a greedy based algorithm is developed. The algorithm sorts the requested resources in ascending availability order (i.e. depending on how many possess a certain competence) in one list, L^r, where the most scarce competences are found at the top of the list. Another list, L^a, is created for the available personnel, sorted according to their expected response time to the accident site. If multiple firefighters have the same response time, e.g. if they are located at the same station, they are sorted in ascending order based on the number of competences that each person possess. Then starting from the top of L^r, the first matching element in L^a is assigned to the accident, given that there are available vehicles that can transport the person. Requested vehicles are assigned based on response time, while checking that there are available firefighters with the right competence to drive them.

Input data and test setting

To investigate the quality of the heuristic, and later to validate the preparedness measure, a test setting was developed. This was based on a real region in Denmark, including two medium sized cities and around ten smaller communities, which corresponds to a medium sized real FRS organization in Sweden. In the validation phase, FRS experts were expected to evaluate the preparedness in the area. Therefore, a Danish area was selected, to get something similar to Sweden without risking that the participating experts had local knowledge that could skew the results. The area was divided into 2584 zones. Travel times between zones were based on Open Street Map data (http://www.openstreetmap.org/) and generated using Network Analyst in ArcMap 10. The expected number of accidents in each zone for each accident type was based on values for similar areas in Sweden, and validated as reasonable by experts from the FRS. All zones in the test setting do not have an expected number of accidents for all three types of accidents. There are 2282 zones with expected building fire accidents, 2375 with expected traffic accidents and 99 zones with expected fires in high rise buildings. This gives a total of 4756 problems (one for each zone and accident type with a positive demand) that have to be solved for the preparedness to be fully measured for the test setting.

Figure 2. An extract of the map of the test area

A total of 44 vehicles of seven different types were included. Furthermore, the setting included 61 firefighters (persons in the model) that could have one or more of six personnel competences; most had between two and three. The test setting includes a total of nine fire stations. In Figure 2, three of these are shown, and numbered 20, 26 and 24. 26 and 24 are part time fire stations and number 20 is a full time fire station.

The preparedness was calculated for three different types of accidents - traffic accidents, building fire accidents and for high rise building fire accidents. The required resources for each accident are presented in Table 1. Even if a maximum of two vehicle types and four competences are required, additional personnel might be required to drive the requested vehicles (since all persons are not permitted to drive all vehicles) and vehicles of types that are not requested might be used to transport the required personnel.

Table 1. Resource requirements for the different types of accidents

Type of accident	Requested competences	Requested vehicle types
Traffic accidents [TA]	4 x Basic firefighting, 1 x Team leadership	1 x Base vehicle
Building fires [BF]	3 x Basic firefighting, 1 x BA operation leadership, 1 x Team leadership,	1 x Base vehicle
Fires in high rise buildings [HBF]	3 x Basic firefighting, 2 x Ladder vehicle operation, 1 x BA operation leadership, 1 x Team leadership	1 x Base vehicle, 1 x Ladder vehicle

Based on the test setting, 22 different scenarios were constructed, with the aim of capturing different operational situations. Therefore, in each scenario of the test setting, the type and number of available resources, as well as their location, is different. Resources can for example be busy handling accidents or non-urgent tasks.

Quality of the heuristic versus exact solutions

To investigate if the heuristic was adequate to use for selecting resources to calculate the preparedness, solutions produced by the heuristic were compared against exact solutions produced by Gurobi Optimizer 5.6.0, for the 22 different scenarios. For each scenario, the resource selection model (see Appendix A), had to be solved for each zone, for all three types of accidents, i.e. 4756 times.

The computational results were obtained using a PC laptop, with an Intel Core i5-2520M, 2.50 GHz processor, and 8 Gb RAM. The solution time for solving the model for one scenario, using Gurobi varied between 2 and 10 hours. The corresponding solution time for the heuristic varied between 7 and 10 seconds for the different scenarios.

Table 2 presents aggregated results for each scenario, where optimal and heuristic solutions are found for all accident types and zones. In most cases the heuristic finds an optimal solution, as indicated by the fourth column in Table 2, which displays the number of times (out of 4756) the heuristic fails. In 12 of the 22 scenarios, the heuristic always finds an optimal solution. In Scenario 2 and 12, it delivers inferior solutions in 37-50% of the problems solved. However, looking at the difference in mean objective function values (expressed as the mean preparedness values; columns 5-7), it is only in Scenario 2 that the solutions produced by the heuristic are more than 1% worse than the optimal solution on average. The reason for this is probably that Scenario 2 has few available resources, located in a manner that makes it difficult for the heuristic to perform optimal selections.

In total, 104 632 problems were solved heuristically and exactly. In over 95% of the cases, the heuristic found an optimal solution. In the rest of the cases, the solutions were never more than 31% from the optimal. Thus, we conclude that the heuristic is good enough to be used for computing the preparedness measure, at least for the specific test setting used here.

Table 2. Computational results comparing the heuristic to optimal solutions

Scenario	Number of firefighters	Number of vehicles	Number of problems where the objective function values differ	Mean preparedness for optimal solutions	Mean preparedness for heuristic solutions	Difference [%]
1	50	13	0	7.610	7.610	0.00%
2	14	6	2383	13.435	12.610	6.54%
3	26	7	33	10.385	10.369	0.15%
4	20	8	0	9.483	9.483	0.00%
5	23	8	18	9.092	9.085	0.07%
6	34	9	4	8.557	8.557	0.00%
7	33	9	86	8.722	8.717	0.05%
8	14	5	0	10.719	10.719	0.00%
9	18	5	0	11.540	11.540	0.00%
10	25	8	0	9.458	9.458	0.00%
11	28	11	0	8.786	8.786	0.00%
12	25	8	1769	10.547	10.510	0.36%
13	37	11	0	7.987	7.987	0.00%
14	24	10	0	9.173	9.173	0.00%
15	32	9	91	8.796	8.794	0.02%
16	27	8	0	9.530	9.530	0.00%
17	29	9	0	9.814	9.814	0.00%
18	23	7	0	10.878	10.878	0.00%
19	33	9	20	8.085	8.084	0.01%
20	15	6	0	11.332	11.332	0.00%
21	27	9	171	9.404	9.396	0.09%
22	18	7	253	11.598	11.565	0.29%

CALIBRATION AND VALIDATION OF THE PREPAREDNESS MEASURE TO MATCH SUBJECTIVE OPERATIONAL EXPERTISE

To be able to adequately utilize the preparedness calculations, for example as a support tool in different planning situations, the preparedness value has to be calibrated and validated for the specific area. Here, this was done for the test setting using preparedness evaluations made by professional staff from different FRS in Sweden.

The Evaluation Process

In the evaluation process, the 22 scenarios in the test setting have been evaluated by FRS commanders selected by six participating FRS organizations. They had varying experience from command-and-control work where similar preparedness evaluations are common, between zero and more than 30 years. Therefore, each scenario was evaluated by at least two experts. For each scenario, the preparedness has been graded for 13 selected zones (see Figure 2 where seven of the 13 zones are shown, and marked as A-G). The assignment for the experts (i.e. the FRS personnel) was to evaluate the preparedness in each of the 13 zones, by coloring the zones red for inadequate preparedness, yellow for acceptable, and green for good preparedness. They had to take into account which resources that were available, and where these were located. The experts were provided with maps visualizing the number of expected accidents as well as expected travel times in the area. In total, 623 evaluations were made.

A model for finding thresholds

One purpose with the calibration process was to find thresholds for the preparedness measure, so that is it possible to say if the calculated values correspond to good or inadequate preparedness. To find such thresholds, an integer programming model with the objective of minimizing the deviation between the calculated preparedness and the perceived preparedness was constructed and solved.

The objective of the model is to find thresholds *R* and *S*, so that the number of evaluations where the preparedness as perceived by the firefighters differs from the calculated value is minimized. A calculated preparedness value above *R* indicates that the preparedness is inadequate, while a value below *S* means that the preparedness is good. Values between *R* and *S* correspond to acceptable preparedness.

The evaluations made by the FRS personnel were gathered into three sets, I, A and G, where each evaluation corresponds to one assessment of one zone by one person for one scenario. Depending on the assessment, the evaluation will end up in one of the sets; e.g. if the expert deems the preparedness to be acceptable, the evaluation belongs to set A and thus should have a calculated preparedness value between R and S. The complete model can be found in Appendix B.

Results from the calibration

Of the 623 evaluations, 208 were perceived as inadequate, 208 as acceptable and 207 as good. This in itself is an interesting result. The scenarios were selected to get a varying mix where some scenarios had many and some had fewer available resources, giving various states of preparedness. But given that the evaluations are subjective and done by multiple individuals, it is unexpected that the evaluations would be so evenly distributed.

The results from the evaluations compared to the calculated preparedness values are presented in Figure 3. Each set is represented by a line, and have been sorted in increasing order, i.e. the dotted green line shows the elements in the set G, with corresponding calculated values on the vertical axis. That is, one point making up the green line matches one evaluation made by one expert, who considered the preparedness good. As can be seen in the figure, there is a significant overlap between the sets, making it impossible to find thresholds that do not give any mismatch between perceived and calculated preparedness. One possible explanation might be that the sample is from operational staff from different areas of Sweden, with various conditions for response operations in their own area. Thus, even if the experimental setting is the same for all participants, the local characteristics of their home area might affect their judgements.

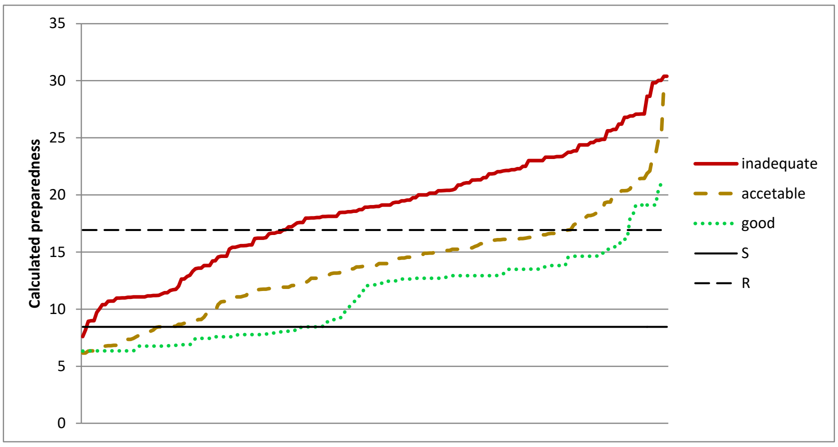

Figure 3. Results from comparing calculated with perceived preparedness

The model was solved using Frontline Systems' Analytic Solver Platform in MS Excel. An optimal solution was produced in less than a minute with an objective function value of 271. The optimal thresholds, S = 8.45, R = 19.93, give a result where there is a match between the perceived and the calculated preparedness in 367 occasions out of 623. One interpretation of this is that there is a 60% chance that an expert will agree with the calculations made by the preparedness measure, if the optimal thresholds are used. Of course, it would be desirable to have a higher matching rate, but unfortunately, the individuals are not consistent with each other in their evaluations, even when judging the same scenario (a result matched by the one reported in Andersson Granberg (2013)).

An example of this is shown in Figure 4, where the evaluated scenario is when all resources are available and located at their home stations. In two of the 13 zones, all 28 experts, as well as the calculated measure, agree that the preparedness is good. In the rest of the zones, there are varying levels of disagreement, which are especially noteworthy in zone 1, 3, 4 and 7, where some experts deem the preparedness to be good, while other experts consider it to be inadequate. In this particular scenario, the measure tends to calculate the preparedness as slightly worse than a majority of the experts (in zone 1, 2, 5, 7 and 9). This was partly explained by comments from some of the experts during the evaluations, that "if all resources are available at the station, the preparedness must be good".

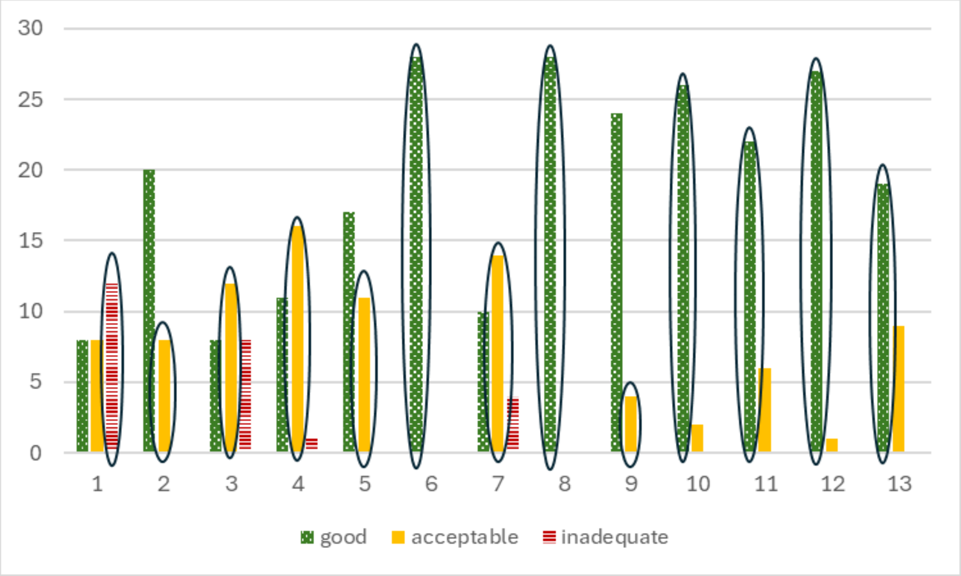

Figure 4. Perceived preparedness by 28 experts for one scenario in colored bars. Calculated preparedness is indicated by the circled bars

The scenario evaluated in Figure 4, is one of the less complex scenarios that the experts had to assess. When the complexity increases, so does the variation in the evaluations. The use of a preparedness measure might possibly help to decrease this variation, and create a common understanding for the preparedness concept.

To further investigate the expert disagreement, an index of diversity was calculated for five scenarios, with the same resource and demand setup, making it possible to compare the expert assessments (Agresti & Agresti, 1978). It is calculated as:

$$Div = 1 - \sum_{i=1}^{3} p_i^2 \tag{2}$$

where p_i is the proportion of evaluations in a certain category (good, acceptable, inadequate). *Div* is calculated for each zone and scenario, giving the results in Figure 5.

Scenarios and zones in Figure 5 are sorted according to increasing average diversity. Averages are noted within parathesis, e.g. the average diversity index over all zones for Scenario 1 (S1) is 0.35. For Zone 4, over all scenarios, it is 0.25. This can be interpreted as the probability that two experts will have different evaluations of the preparedness (Agresti & Agresti, 1978). Thus, in Scenario 3, the experts were quite in agreement with each other regarding the preparedness, even though in Zone 7, 1 and 3, the index is still over 0.5. Zone 1 and 3 (A and C in Figure 2), were the two zones most difficult for the experts to agree on, and are both zones without any station close to it.

It should be noted that the diversity index does not differentiate between evaluations close to each other (like good and acceptable), and far from each other (good and inadequate) but only describes the likelihood that two experts will have exactly the same assessment. Still, it is interesting to note the variability of the index over zones and over scenarios, indicating that some situations are easier to assess than others.

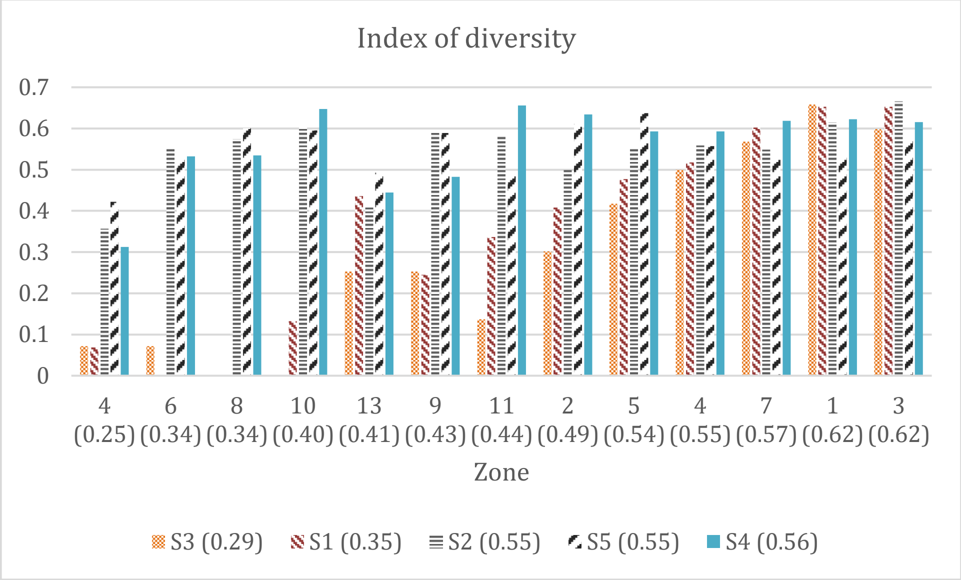

Figure 5. Index of diversity for 28 experts evaluating the preparedness in 13 zones in five scenarios

UTILIZATION OF THE PREPAREDNESS MEASURE

Using the calibrated measure, it is possible to visualize the preparedness, and an example is shown in Figure 6; the preparedness for building fires is calculated and visualized as a heat map for each zone. Vehicle locations in the map are marked by a line ending with a label stating the vehicle id, e.g. 253. Firefighters are represented by the symbols looking like a Θ, where the colors define which competences the person has. Red zones have a preparedness value over 16.93, orange zones between 8.45-16.93, and green zones below 8.45, in accordance with the calibrations made for the test setting.

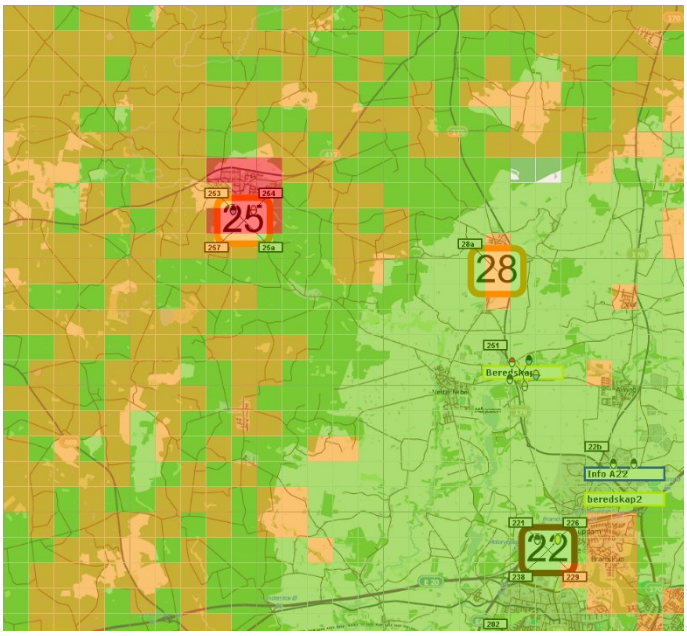

Figure 6. An example of how preparedness can be visualized, where red squares indicate inadequate preparedness, orange squares acceptable preparedness, and green squares good preparedness

As can be seen in Figure 6, most of the area is orange or green, corresponding to acceptable or good preparedness. The red area close to Station 25, can be explained by the fact that there is no base vehicle nearby, even if four other vehicles of different types can be found at the station. Furthermore, there are only two firefighters nearby, while the additional required resources (according to Table 1) have a longer response time.

This kind of map provides a visual overview of the preparedness. It can be used to present the current situation in an area and to facilitate the operational planning process. When the dispatcher continuously can visualize the preparedness and all the resources in the current area, it is possible to visually identify whether any type of resource is missing or if a resource has to be moved. If the preparedness has deteriorated to an unacceptable level in a zone, the dispatcher can utilize the visualization to determine potential actions to improve the preparedness level, e.g. by locating resources closer to the affected zone. Such decisions can also be tested and evaluated before an actual decision is made.

If there had been a wide agreement on the preparedness measure, then we could have concluded that this basic heat map visualization is sufficient. However, our results indicate that this level of visualization might be insufficient, since there is disagreement on how to evaluate preparedness. Thus, this motivates further development of more advanced preparedness dashboards, both for adjusting the measures per se, and to generate a deeper understanding of its basis. Such an interface could be used to develop the measure further together with stakeholders, during training, and during operations.

CONCLUSION

In this paper, the development and validation of a preparedness measure for Swedish fire and rescue services is described. An integer programming model has to be solved to calculate the measure, and solutions are quickly found by a heuristic. The heuristic solutions are optimal in more than 95% of the cases. When comparing this estimate to the subjective judgements by professional staff, experts agree with the measure in about 60% of the cases. The results also show that the experts do not agree with each other all the time, especially when the preparedness as calculated by the measure is inadequate or acceptable (i.e. not good). This makes it impossible to calibrate the measure so that it will always give a result that corresponds to the evaluations. Thus, the preparedness as perceived by the fire and rescue service personnel is an ambiguous concept, and opinions about what constitute good or inadequate preparedness might vary even between close colleagues.

A preparedness measure like the one developed here might help to reduce this vagueness. If implemented in a geographical information system, it should become clear at least how the measure calculates the preparedness, and in time, the users might adopt this view. Thus, the measure could facilitate a common ground for situational awareness between individual decision makers. Alternatively, if harmonization of subjective judgments across a larger geographical area is not desired, local measures can be developed. These results are critical for further digitalization, centralization and virtualization of response command centers. They show that the individuals are not interchangeable, i.e. someone from a different area may have a different view of what is sufficient. However, the visualization could be made more advanced than the current heat map, e.g. including information on resource requirements and accident frequencies.

It is possible to directly use the measure as a base for dispatching and relocation decisions. This, just by observing where the preparedness is good or inadequate, and refrain from moving resources away from areas with an inadequate preparedness, while opting for resources close to areas with good preparedness when dispatching or relocating. The next step is to incorporate the measure in automated decision support tools that can suggest which resources to dispatch or to relocate, based on expected response times and preparedness levels.

For future research, it would also be interesting to include stochastic aspects in the measure, examples being stochastic travel times and varying resource availability. As this will make the associated optimization problem harder to solve, new exact solution procedures should also be explored. These will most likely be too slow for our purposes, so heuristics, or hybrid approaches, are probably necessary as well. It would also be interesting to establish the theoretical approximation ratios for the heuristics, especially if it is possible to better match the preparedness measure to the FRS experts' perceptions. It is an interesting challenge, especially considering that preparedness will always be affected by uncertainties and subjectivity. A preparedness value calculated by a heuristic can thus be closer to the perception of one decision maker, than the value produced by an exact solver. Furthermore, the usefulness of a preparedness measure also depends on how it is presented to a decision maker. Heat maps are simple and easy to understand, but limited in how much information they can communicate, so combining the development of the mathematical measure with the development of the information visualization is to be recommended. Connected to a qualitative assessment of how the experts would use this kind of decision support tool, it would also be interesting to delve deeper into which factors they consider when evaluating the preparedness, and try to incorporate these.

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APPENDIX A: THE RESOURCE SELECTION MODEL

Sets:

A = set of accidents
J = set of zones
U = set of vehicles
P = set of persons
K = set of vehicle types
R = set of personnel competences

Variables:

$$x_{praj} = \begin{cases} 1, \text{if person } p \text{ contributes with competence } r \text{ to accident } a \text{ in zone } j \\ 0, \text{otherwise} \end{cases}$$

$$y_{uaj} = \begin{cases} 1, \text{if vehicle } u \text{ contributes to accident } a \text{ in zone } j \\ 0, \text{otherwise} \end{cases}$$

$$z_{puaj} = \begin{cases} 1, \text{if person } p \text{ is assigned to vehicle } u \text{ to accident } a \text{ in zone } j \\ 0, \text{otherwise} \end{cases}$$

$$f_{puaj} = \begin{cases} 1, \text{if person } p \text{ drives vehicle } u \text{ to accident } a \text{ in zone } j \\ 0, \text{otherwise} \end{cases}$$

V_{paj} = response time for person p to accident a in zone j (if the person is requested)
T_{uaj} = response time for vehicle u to accident a in zone j (if the vehicle is requested)

Parameters:

$$E_{pu} = \begin{cases} 1, \text{if person } p \text{ can be assigned to vehicle } u \\ 0, \text{otherwise} \end{cases}$$

$$G_{pu} = \begin{cases} 1, \text{if person } p \text{ can drive vehicle } u \\ 0, \text{otherwise} \end{cases}$$

$$Q_{pr} = \begin{cases} 1, \text{if person } p \text{ posesses competence } r \\ 0, \text{otherwise} \end{cases}$$

$$S_{uk} = \begin{cases} 1, \text{if vehicle } u \text{ is of type } k \\ 0, \text{otherwise} \end{cases}$$

W_u = number of seats in vehicle u
t_{uj} = travel time for vehicle u to zone j
d_{aj} = expected number of accidents of type a in zone j
τ_p = preparation time for person p
B_{ra} = number of persons with competence r required to accident a
D_{ka} = number of vehicles of type k required to accident a

$\forall\ a \in A, j \in J$:

$$\mathit{Minimize} \quad \sum_{p \epsilon P} V_{paj} + \sum_{u \epsilon U} T_{uaj} \tag{3}$$

Subject to

$$\sum_{p \in P} x_{praj} \geq B_{ra} \qquad \forall\ r \in R \tag{4}$$

$$\sum_{u \in U} S_{uk}\, y_{uaj} \geq D_{ka} \qquad \forall\ k \in K \tag{5}$$

$$\sum_{p \in P} z_{puaj} \geq y_{uaj} \qquad \forall\ u \in U \tag{6}$$

$$\sum_{p \in P} z_{puaj} \leq W_{ua} \sum_{p \in P} f_{puaj} \qquad \forall\ u \in U \tag{7}$$

$$\sum_{u \in U} z_{puaj} \geq \sum_{r \in R} x_{praj} \qquad \forall\ p \in P \tag{8}$$

$$\sum_{u \in U} z_{puaj} \leq 1 \qquad \forall\ p \in P \tag{9}$$

$$\sum_{r \in R} x_{praj} \leq 1 \qquad \forall\ p \in P \tag{10}$$

$$x_{praj} \leq Q_{pr} \qquad \forall\ p \in P, r \in R \tag{11}$$

$$z_{puaj} \leq E_{pu} \qquad \forall\ p \in P, u \in U \tag{12}$$

$$\sum_{p \in P} f_{puaj} \leq 1 \qquad \forall\ u \in U \tag{13}$$

$$\sum_{u \in U} f_{puaj} \leq 1 \qquad \forall\ p \in P \tag{14}$$

$$f_{puaj} \leq z_{puaj} \qquad \forall\ p \in P, u \in U \tag{15}$$

$$f_{puaj} \leq G_{pu} \qquad \forall\ p \in P, u \in U \tag{16}$$

$$T_{uaj} \geq \left(t_{uj} + \tau_p\right)\left(z_{puaj} + y_{uaj} - 1\right) \qquad \forall\ u \in U, p \in P \tag{17}$$

$$V_{paj} \geq t_{uj}\left(z_{puaj} + \sum_{r \in R} x_{praj} - 1\right) + \tau_q\left(z_{puaj} + \sum_{r \in R} x_{praj} + z_{quaj} - 2\right) \qquad \forall\, p,q \in P, u \in U \qquad (18)$$

$$x_{praj},\, y_{uaj},\, z_{puaj}, f_{puaj}, E_{pu}, Q_{pr},\, S_{uk}, G_{pu} \in \{0,1\} \qquad \forall\, p \in P, r \in R, u \in U, k \in K \qquad (19)$$

$$T_{uaj}, V_{paj} \;\geq 0 \qquad \forall\, u \in U, p \in P \qquad (20)$$

The objective function (3) is the sum of the response times for the selected vehicles and personnel. Constraint (4) ensures that enough personnel with the right competences, and (5) that the requested vehicles, are selected. Constraint (6) states that there always must be at least one person assigned to a vehicle for the vehicle. Constraint (7) ensures that the vehicle seating capacity is not exceeded and that a driver is assigned to each vehicle. There must be a vehicle available to transport each person to the accident site, which is ensured by Constraint (8). Furthermore, a person can only be assigned to maximum one vehicle at a time, only one competence can be utilized at a time, and a person can only contribute with a certain competence if (s)he possesses the competence. These restrictions are ensured by Constraint (9), (10) and (11). Constraint (12) guarantees that a person only can be assigned to a vehicle if s(he) is available and permitted to be assigned to that vehicle. A person can for example only be assigned to a vehicle if they are located at the same place. That a vehicle can only be driven by one person at a time, and that a person only can drive one vehicle at a time is ensured by Constraint (13) and (14) respectively. The person that drives a vehicle must also be assigned to that vehicle, i.e. the person travels in that vehicle to the accident site, which is ensured by Constraint (15). Constraint (16) ensure that a person has the specific skills to drive a certain vehicle. Constraint (17) set the response time for vehicles that are required for the accident work (vehicles that are only used for transporting personnel will get a response equal to zero, since they should not contribute to the objective function value). Thus, T_{uaj} will be non-zero only if y_{uaj} is equal to one. The response time is calculated as the travel time for the vehicle plus the preparation time for the person assigned to the vehicle with the longest preparation time. Therefore, the constraint set is defined for all personnel, and T_{uaj} is set to the largest of the right hand side values, where both z_{puaj} and y_{uaj} are equal to one. In a similar manner as for (17), Constraint (18) set the response time for personnel required for accident work, as the travel time for the vehicle the person is assigned to, plus the preparation time for the person assigned to the same vehicle that has the longest preparation time.

APPENDIX B: THE MODEL FOR FINDING THE THRESHOLDS

The objective of the model is to find thresholds *R* and *S*, so that the number of evaluations where the preparedness as perceived by the firefighters differs from the calculated value is minimized. The evaluations made by the FRS personnel were gathered into three sets, I, A and G, e.g. if the expert deems the preparedness to be acceptable, the evaluation belongs to set A. For each evaluation, the preparedness measure was used to calculate a value, giving the parameters *r*, *s* and *t*.

Sets:

I = set of evaluations where the preparedness is considered inadequate

A = set of evaluations where the preparedness is considered acceptable

G = set of evaluations where the preparedness is considered good

Variables:

R = threshold for when the preparednss is considered inadequate (if exceeded)
S = threshold for when the preparednss is considered good (if below)

$$x_i^{IR} = \begin{cases} 1, & \text{if evalution } i \text{ has a calculated value below the threshold } R \\ 0, & \text{otherwise} \end{cases}$$

$$x_i^{IS} = \begin{cases} 1, & \text{if evalution } i \text{ has a calculated value below the threshold } S \\ 0, & \text{otherwise} \end{cases}$$

$$x_j^{AS} = \begin{cases} 1, \text{ if evalution } j \text{ has a calculated value below the threshold } S \\ 0, \text{ otherwise} \end{cases}$$

$$x_j^{RA} = \begin{cases} 1, \text{ if evalution } j \text{ has a calculated value above the threshold } R \\ 0, \text{ otherwise} \end{cases}$$

$$x_k^{SG} = \begin{cases} 1, \text{ if evalution k has a calculated value above the threshold } S \\ 0, \text{ otherwise} \end{cases}$$

$$x_k^{RG} = \begin{cases} 1, \text{ if evalution } k \text{ has a calculated value above the threshold } R \\ 0, \text{ otherwise} \end{cases}$$

Parameters:

r_i = calculated preparedness value for evaluation $i \in I$

s_j = calculated preparedness value for evaluation $j \in A$

t_k = calculated preparedness value for evaluation $k \in G$

M = a large number

$$Minimize \quad \sum_{i \in I} (x_i^{IR} + x_i^{IS}) + \sum_{j \in A} (x_j^{AS} + x_j^{RA}) + \sum_{k \in G} (x_k^{SG} + x_k^{RG}) \tag{21}$$

$$Subject\ to$$

$$R - r_i \leq M x_i^{IR} \qquad \forall\, i \in I \tag{22}$$

$$S - r_i \leq M x_i^{IS} \qquad \forall\, i \in I \tag{23}$$

$$S - s_i \leq M x_j^{AS} \qquad \forall\, j \in A \tag{24}$$

$$s_i - R \leq M x_j^{RA} \qquad \forall\, j \in A \tag{25}$$

$$t_k - S \leq M x_k^{SG} \qquad \forall\, k \in G \tag{26}$$

$$t_k - R \leq M x_k^{RG} \qquad \forall\, k \in G \tag{27}$$

$$x_i^{IR}, x_i^{IS}, x_j^{AS}, x_j^{RA}, x_k^{SG}, x_k^{RG} \in \{0,1\} \qquad \forall\ i \in I, j \in A, k \in G \tag{28}$$

$$R, S\ \geq 0 \tag{29}$$

The x-variables take the value 1 each time that there is a mismatch between a calculated value and a perceived value. E.g. in (22), if the value for r_i is below the threshold R, the variable x_i^{IR} is forced to take the value 1. Lowering R to compensate for this might cause problems in e.g. (25) where an evaluation that the preparedness is good, might get a calculated value above the threshold, which is not desired. Each evaluation is compared to the two thresholds, giving the x-variables and the constraints (22)-(27). The objective function (21) is the sum of the x-variables, and the objective function value is thus the number of times where the perceived and the calculated values do not correspond, plus an extra penalty for the occasions when the calculated value is on the wrong side of both thresholds (e.g. if x_k^{RG} is one, then x_k^{SG} will also be one, since t_k cannot be higher than R without being higher than S).